



EFRE INTERREG AMOR PROJECT ATCZ203

Interference Analysis, Modeling, and Emulation

Research Group Embedded Systems
FACHHOCHSCHULE OBERÖSTERREICH

Advisor:

Prof. Dr. Ing. habil. Hans Georg BRACHTENDORF

by

M.Sc. Martin STEIGER

Prof. Dr. Ing. habil. Hans Georg BRACHTENDORF

Dr. Kai BITTNER

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Abstract

The representation of linear components and circuits in the frequency domain using S-parameters is a common industry standard in radio frequency engineering. Linear mappings (i.e. matrices) of incoming power waves A to outgoing power signals B are defined for selected frequency ranges. However, real non-linear effects of components (e.g. the IP3 in amplifier circuits) cannot be modelled satisfactorily in this way. Therefore, some time ago Agilent introduced a new standard referred to as X-parameters, which models the influences of harmonic distortion at the input for pre-defined harmonics. The extraction of these so-called X-parameter models can be done in several different ways. Already functional components can be measured very efficiently with an NVNA (non-linear vector network analyser). Most NVNAs have this functionality already integrated due to a manufacturer agreement for a common standard. Alternatively, existing and sufficiently accurate component models (differential equation systems, etc.) can be used. How an X-parameter model can be generated from such models will be explained in more detail in this paper. In addition, the results are compared with common multi-tone harmonic balance analysis of the underlying circuit differential equations.

1 X-Parameter Fundamentals

As in the case of the S-parameters, a consideration of the X-parameters as power waves ratios in the frequency domain is the simplest approach to begin with. One distinguishes between the incoming waves of an N-port A_p , $p = 1, \dots, N$ and the scattered waves B_p at port p (which both may be calculated using the appropriate voltage and current ratios at p). In contrast to the S-parameter models mentioned above, however, a non-linear mapping rule is now applied in the frequency domain to take the harmonic distortions into account. Assuming that the incoming time domain signal A_p is periodic, it can be represented as a Fourier series with a fixed fundamental frequency f_0 (double sided spectre):

$$a_p(t) = \sum_{k \in \mathbb{Z}} a_{p,k} \cdot e^{j(k\omega_0 t + \phi_k)}, \omega_0 = 2\pi f_0 \quad (1)$$

$$A_p(f) = \sum_{k \in \mathbb{Z}} \underline{a_{p,k}} \cdot \delta(f - kf_0), \underline{a_{p,k}} = \underline{a_{p,-k}^*}$$

The same applies, evidently, to the scattered waves B . For further considerations let $A_{p,k} = \underline{a_{p,k}}$ and $B_{p,k} = \underline{b_{p,k}}$ be the coefficients of the k th harmonic of the Fourier series of the input respectively output signal at port p . A graphical representation is given in Figure 1, where A represents the stimulus of the given component. Since we are now looking for a model for the mapping $A \Rightarrow B$, similar to the S-parameters, we need a mapping rule which describes the influence of the incident harmonics at all ports on a single harmonic of the scattered wave at a particular port p .

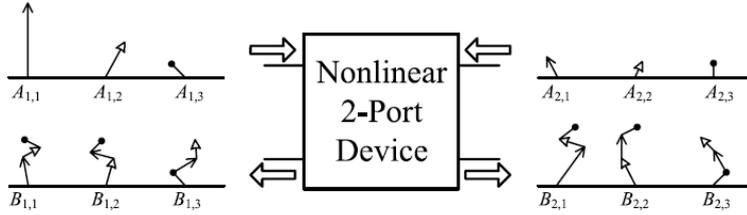


Figure 1: Non-Linear 2-Port Component Model [1]

Let the phase information of the fundamental harmonic of the input signal A be known as $P = e^{j \cdot \arg(A_{1,1})}$. The actual derivation of the X-parameter formalism is omitted here. For further research please refer to [2]. Let the following representation of the non-linear mapping function $B = F(A)$ (known from poly-harmonic distortion (PHD) modelling [2]) be used:

$$\begin{aligned}
B_{(p,k)} &= X_{p,k}^{(F)}(|A_{1,1}|, DC, f_0) \cdot P^k \\
&+ \sum_{\substack{q=1, l=1 \\ (q,l) \neq (1,1)}}^{q=N, l=K} X_{p,k,q,l}^{(S)}(|A_{1,1}|, DC, f_0) \cdot A_{q,l} \cdot P^{k-l} \\
&+ \sum_{\substack{q=1, l=1 \\ (q,l) \neq (1,1)}}^{q=N, l=K} X_{p,k,q,l}^{(T)}(|A_{1,1}|, DC, f_0) \cdot A_{q,l}^* \cdot P^{k+l}
\end{aligned} \tag{2}$$

The formalism here contains numerous different indices and additional variables, which are now explained

- p, q : port indices
- k, l : harmonic indices
- $X_{(p,k)}^{(F)}$: the influence of the fundamental harmonic $A_{(1,1)}$ at port 1 on the k -th harmonic of the scattered wave $B_{(p,k)}$ at port p
- $X_{(p,k,q,l)}^{(S)}$: the influence of the l -th harmonic of the incident wave $A_{(q,l)}$ at port q on the k -th harmonic of the scattered wave $B_{(p,k)}$ at port p
- $X_{(p,k,q,l)}^{(T)}$: the influence of the l -th harmonic of the conjugate incident wave $A_{(q,l)}^*$ at port q on the k -th harmonic of the scattered wave $B_{(p,k)}$ at port p .

It is noticeable that the influence of the fundamental harmonics at port 1 is not normalized by $A_{(1,1)}$. This is because the three components of the X-parameters are already normalized by $A_{(1,1)}$ (for example, the magnitude of the fundamental harmonics must be included when generating X-parameter models in AWR). In corresponding X-parameter files, so-called XnP files, this scaling information is also included. As a brief side note, X-parameter models formulated in this way are mostly used in combination with harmonic balance (HB) analyses. From the X-parameters, a Volterra series can be generated, which allows circuit analysis using simple HB methods. However, these Volterra models are not part of this report, which focuses on generating X-parameter representations from existing models and comparing it with HB results.

2 Using the AWR Design Environment to Generate X-Parameter Models

Among the essential aspects of this work, the evaluation of already existing features of X-parameter modelling from device simulations shall be included.

For this purpose, the AWR Design Environment will be considered in more detail. This is a commercial tool from ©Cadence, which is used in particular for the simulation of radio frequency devices and comprehensive electronic design automation. This software consists of several sub-components, such as Microwave Office for radio frequency circuit systems and AXIEM/Analyst for EM field simulations. An overview of the features is provided in Figure 2.

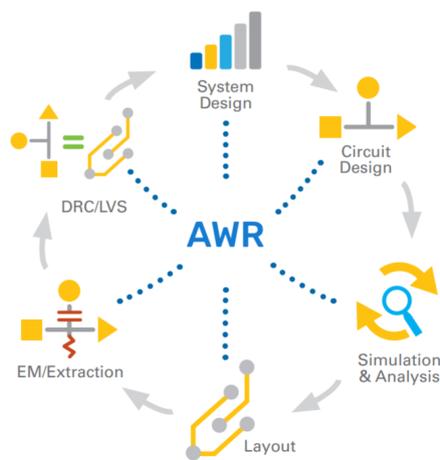


Figure 2: AWR Software Platform

However, relevant for our purposes are mainly the methods of non-linear circuit analysis such as several variants of the harmonic balance method. The default simulation engine for PHD modelling is the single-tone harmonic balance technique. However there exist other time or frequency domain algorithms for non-linear modelling. These variants are not considered further, but the fundamentals have already been evaluated in the context of the ARMOR project.

2.1 PHD Model Generator

As is already mentioned, X-parameters are essentially based on the poly-harmonic distortion (PHD) modelling from [2]. AWR provides a PHD model generator that can automatically generate such models (which are called XPARAM blocks in AWR) from existing schematics without the need for a separate measurement setup. However, since the implementation of this method is not publicly available, as is almost always the case with commercial tools, we can only make assumptions based on the adjustable parameters. What is clear, however, is that a harmonic balance technique is used. The modifiable parameters are (based on the provided user guide)

- **Circuit Schematic:** selected circuit schematic to be modelled as a XPARAM block.

- **Data File Name:** name of the XnP file that is generated based on circuit. schematic measurements. The structure is described in Chapter 2.2
- **Maximum Mixing Order:** determines the maximum allowable frequencies of the truncation scheme of a HB run. E.g. when a maximum order of 10 is selected for a two-port device, only 5 harmonics are stored.
- **Tone Settings:** specifies the independent excitation frequencies respectively the fundamental frequencies of the X-parameter extraction method. Considering equation (2), it should be mentioned that a separate X-parameter model is created for each selected fundamental frequency. If modelling over a frequency range is required, a grid of fundamental frequencies must be considered according to the desired resolution. An example is displayed in Figure 3, where X-parameter models have been generated from a linear S-Parameter model of a 2.4GHz WLAN bandpass filter from 400 MHz to 4 GHz fundamental frequency with 20 and 50 steps.
 - **Sweep:** If this option is not selected, only one fundamental frequency is taken into account. Otherwise a range for a sweep of the fundamental frequencies can be defined. The higher the number of steps, the more accurate the resulting X-parameter model will be in the given range.
 - **Frequency Selection Mode:** one feature is the use of multi-tone harmonic balance methods with 2 or more fundamental frequencies. This requires a sophisticated truncation scheme. To this end, the fundamental frequencies f_1 and f_2 (two-tone case) must be swept within a frequency range. How those sweeps are joined together is determined with the Frequency Selection Mode. Either *all combinations* (AC) determines the combinations of all occurring tones or *coupled* (C) determines the tones independently. For example considering two frequency sweeps f_1 and f_2 :

$$f_1 = \{f_{11}, f_{12}, f_{13}\}$$

$$f_2 = \{f_{21}, f_{22}, f_{23}\}$$

$$\text{AC: } \{[f_{11}, f_{21}], [f_{11}, f_{22}], [f_{11}, f_{23}], [f_{12}, f_{21}], \dots\}$$

$$\text{C: } \{[f_{11}, f_{21}], [f_{12}, f_{22}], [f_{13}, f_{23}]\}$$

- **Ports From Schematic:** In order to generate an X-parameter model from a circuit, terminals/ports must be selected. Terminal ports have a pre-defined impedance which is by default 50 Ω . The assignment of

these ports is defined here, whereby special attention must be paid to where port 1 is located. By default, port 1 is the input port where the large-signal tone is applied. This determines the $X_{(p,k)}^{(F)}$ parameters. Generally there are three different options for the port assignments:

- **Source:** This is a harmonic signal source with parametrizable source impedance (by default 50Ω) and power at the fundamental harmonic in dBm. It is theoretically possible to include higher harmonics as well, but this is not currently considered in this paper. If one defines multiple frequency sweeps in the *Tone Settings*, the corresponding source powers can be modified here as well. What this port is explicitly **not** to be used for is to feed a small-signal perturbation into the circuit, as one does with the original time- and frequency-domain parameter extraction methods. This happens internally and must not be defined explicitly. It is also worth mentioning that a DC bias may also be specified.
- **Load:** this is a simple port with variable and frequency-dependent termination impedance. It can be defined for higher harmonics as well and can be provided with a DC offset (even if the latter is less needed at the moment).
- **Bias:** variable bias voltage or current without any extra features.

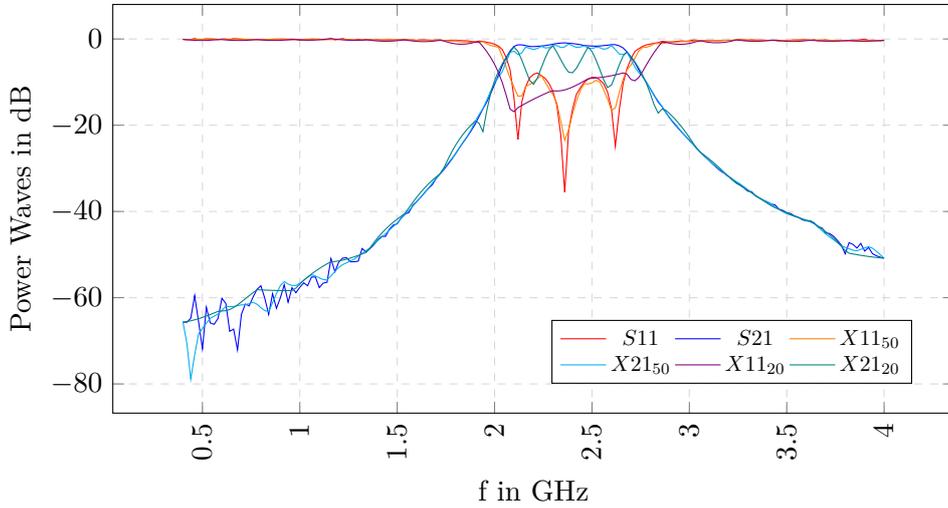


Figure 3: Comparison of the Transmitted and Reflected Wave on Port 1, X-Parameter Models with different Resolutions and S-Parameter Model

2.2 XnP X-Parameter File Format

In this section, the results of the PHD Model Generator in the AWR Design Environment will be discussed. Once an X-parameter model has been generated based on the information in the previous chapter, it is saved in the so called XnP file with a special format. However, this differs in some points from the known Touchstone format for S-parameters, since some additional information must be stored (port selection, power of the large-signal tone source, etc.). Therefore a simple extension of the Touchstone format is not sufficient, but AWR uses the GMDIF (General Measurement Data Interchange Format) as an extension of the MDIF for generalized storage of multi-dimensional data. For further information the ©Keysight reference is located here. There is a uniform convention for the representation of X-parameters, which will be explained in the next paragraphs. For more detailed information, please refer to the previously mentioned link. AWR first generates some header information as a comment. For the actual model, this information is redundant, because it also appears later as variables, but the header increases the readability significantly. A header example is provided here

```
! Generated by APLAC 9.60 Mon Mar 22 15:55:23 2021

! Version 2.0
! HB_MaxOrder = 5
! XParamMaxOrder = 5
! NumExtractedPorts = 2

!-----
! X-Param generation simulation setup
!-----
! fund_1 [4E+08 --> 4E+09], npts=50
! ZX_2_1 [50],ZY_2_1 [0]
! AN_1_1 [0.0316228 --> 0.0316228] (0 dBm --> 0 dBm), npts=1 (BLOCKVAR)
!-----
```

which includes the harmonic balance simulator version (APLAC 9.60), X-Parameter GMDIF-Version (2.0), the maximum number of considered harmonics per fundamental harmonic defined in the frequency sweep, the number of ports, the frequency sweep, characteristic impedance and the power of the large-signal tone input (including possible higher harmonics for multi-tone harmonic balance). The actual relevant section of the file consists of three main components. The **XParamAttributes** contain once again the version, number of ports and the fundamental tones of the large-signal input. In this case, the index column can be neglected, since it is only used for compatibility with the GMDIF format. An example is provided here

```

BEGIN XParamAttributes
% Index(int)      Version(real)      NumPorts(int)      NumFundFreqs(int)
0                2.0                2                1
END

```

The **XParamPortData** block provides reference impedances for the incident and scattered waves at each port of the device under test. As a reminder, AWR still deals with voltage and current signals. To convert these into dependent power quantities (such as the a- and b-waves), one needs the following formulas [1]

$$a_p = \frac{V_p + Z_p \cdot I_p}{\sqrt{8\Re(Z_p)}}, \quad b_p = \frac{V_p - Z_p \cdot I_p}{\sqrt{8\Re(Z_p)}} \quad (3)$$

where Z_p is the port impedance (by default 50 Ω), V_p the port voltage and I_p the current at port p . For the representation in a GMDIF-compliant X parameter file, the following example is given

```

BEGIN XParamPortData
% PortNumber(int)  RefZ0(complex)  PortName(string)
1                50 0    "Port-1"
2                50 0    "Port-2"
END

```

This also includes a column for the port names for the correct matching of the resulting XnP component in the AWR Design Environment. The final block called **XParamData** may actually occur multiple times within an XnP file. To be exact, one block per frequency sweeping point is defined, which is set in the **Tone Settings**. It may also include so-called independent variables, which depend on how the PHD model generator has been set up. Those parameters include the fundamental frequency, the port impedance for the given fundamental, DC offsets, large-signal tone magnitude and so on. Generally, those parameters have got a unified naming convention VAR_ p _ k where p is the port and k is the considered harmonic. For every type of independent variables, there is a defined naming convention which may be revisited here. Additionally there exist dependent variables, which are in the considered case actual X-parameters of type **(FB)**, **(S)** or **(T)** similar to equation (2). There are also parameters for the effects of DC biases etc., but those are currently not considered in this paper. The dependent parameters of interest have got the following naming convention

- **FB_ p _ k** : the wave at the output port p and harmonic k as response of the large-signal tones

- $\mathbf{S}_{p_k_q_l}$: the small-signal added contribution to the wave at port p and harmonic k due to a small-signal incident wave at port q and harmonic l
- $\mathbf{T}_{p_k_q_l}$: the small-signal added contribution to the wave at port p and harmonic k due to a phase-reversed small-signal incident wave at port q and harmonic l .

Since even a simple example cannot be presented here completely due to the amount of data (even a quite rough bandpass model from Figure 3 contains thousands of lines of XParamData blocks), reference is made to the previously mentioned link. It should be mentioned at this point that the X-parameters are generally complex. Complex data are represented GMDIF conform as separate columns in Cartesian coordinates.

One other fact is worth mentioning for practical use. In equation (2), it is noticeable that the DC components are neglected for both incident and scattered waves. This matches with the PHD modelling definition, but the PHD model generator offers the option to define additional DC sources at each device port. A possible application for such a feature is the consideration of the large and small signal behaviour around a given operating point. However, this is not considered in this report and therefore needs to be evaluated separately. Adding the DC sources, however, leads to the fact that additional parameters $XY_{p_q_k}$ appear in the XnP file, which describe the DC effects at port q and harmonic k originating from port p (equation (2) is only extended by the index 0, so A and B contain additional entries regarding the DC components).

2.3 Impedance Matching

Another important aspect needs to be considered when modelling circuits using X-parameters. The PHD model generator requires at least two ports within the circuit, each with a defined terminating impedance (ideally the same). However, if there is a mismatch to this defined impedance in the actual circuit, power losses, reflections etc. will occur. Although gain considerations are primarily relevant when generating the X-parameters, attention must still be paid to this. Especially if for some reason different terminating impedances have to be used.

Consequently, impedance matching has to be applied to each port. There are various solutions available for this, ranging from the variations of transmission line lengths to passive matching circuits. Which method is used depends largely on the application, with power efficiency playing a major role. For the considerations in this report, however, we assume loss-free components. Therefore, we can use simple passive structures such as LC, T or Pi circuits. However, these have the decisive disadvantage that they are designed for one frequency each and can therefore significantly change the

behaviour of the circuit. If there are capacitors at the outputs (for example as DC blockers), an impedance transformation can be performed, which is broadband and thus more suitable for X-parameter modelling. An evaluation of further broadband matching circuits is thus recommended in this context. However, if only a narrow band is of interest, the passive circuits are also valid.

An example of the matching of purely resistive components is given in Figure 4. It must be distinguished whether the source impedance R_S is larger than the load impedance R_L and whether a high-pass or low-pass behaviour is desired. The formulas for the component values L and C at the target frequency f_0 result accordingly:

$$X_L = \begin{cases} R_L \sqrt{\frac{R_S}{R_L - R_S}} & R_L > R_S \\ \sqrt{R_S \cdot R_L - R_L^2} & R_L < R_S \end{cases} \quad (4)$$

$$X_C = R_S \frac{R_L}{X_L}$$

$$C = \frac{1}{2\pi f_0 X_C}, \quad L = \frac{X_L}{2\pi f_0}$$

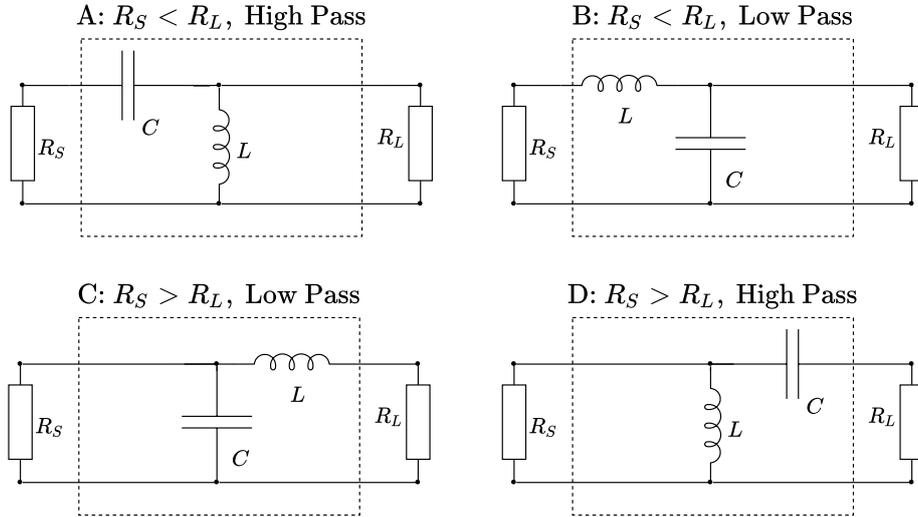


Figure 4: Impedance Matching for Resistive Load Impedances

However, as mentioned above, this type of impedance matching only works for resistive load impedances. Should complex load impedances occur, the imaginary part must be compensated.

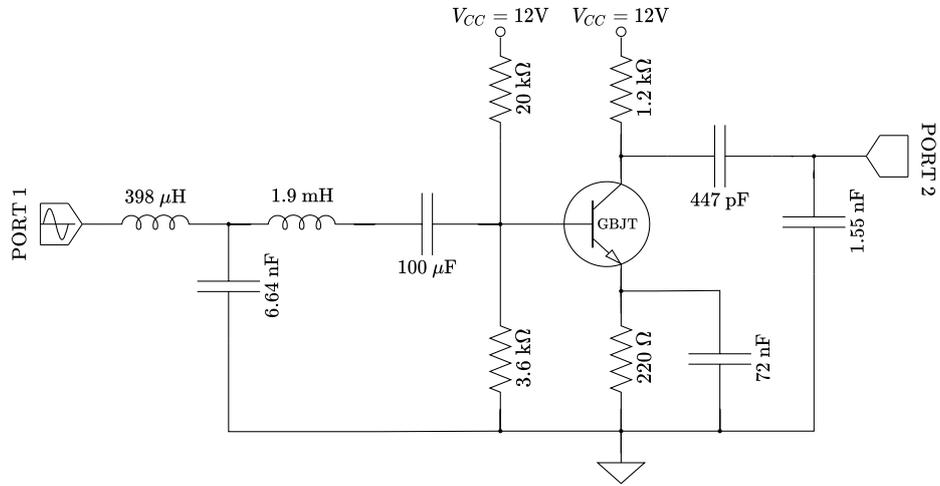


Figure 5: Common Emitter Circuit for $f_0 = 100\text{ kHz}$ and with Impedance Matching to $50\ \Omega$

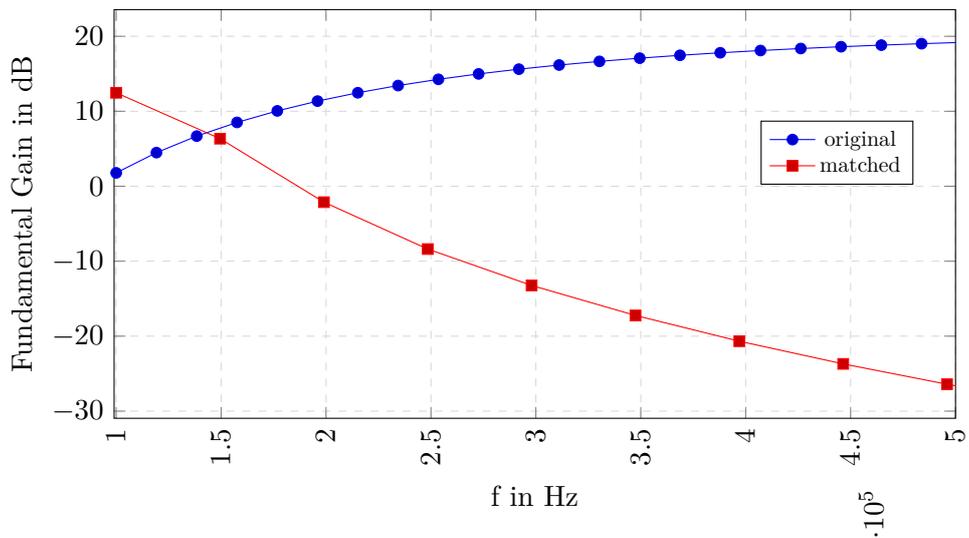


Figure 6: Comparison of the Fundamental Gain at the Original and the Matched Circuit

When using very narrowband matching circuits, however, it must be taken into account that the frequency response of the resulting circuit may change significantly. This is especially applicable for frequency ranges far outside the defined target frequency of the impedance matching. This effect must be taken into account when designing the circuit. It has no influence on

the generation of the X parameter model. An example of a common emitter amplifier with impedance matching is given in Figure 5. In addition, the performance gain of the fundamental harmonics over frequency is given in comparison with a circuit without matching in Figure 6. As expected, the power gains differ greatly here. However, since a common emitter amplifier circuit is generally broadband (only the cut-off frequency can be adjusted via a capacitance at the emitter), other matching structures should be considered. However, it is sufficient for an illustrative example in this report.

To support the properness of the impedance matching and the associated X-parameter model, several X-parameter simulations were conducted. Figure 7 displays the incident and scattered wave powers of the circuit shown in Figure 5, with a power signal source of -30 dBm applied to port 1. The AWR internal APLAC Harmonic Balance solver was used to generate the data at a fundamental frequency of 100 kHz. As can be seen, the measured input power of the incident wave at the fundamental frequency at port 1 matches the power signal source data almost exactly. These deviations result from rounding errors in the dimensioning of the impedance matching and one can therefore assume that the circuit is correctly matched. The reflections of the higher harmonics at port 1 are negligible.

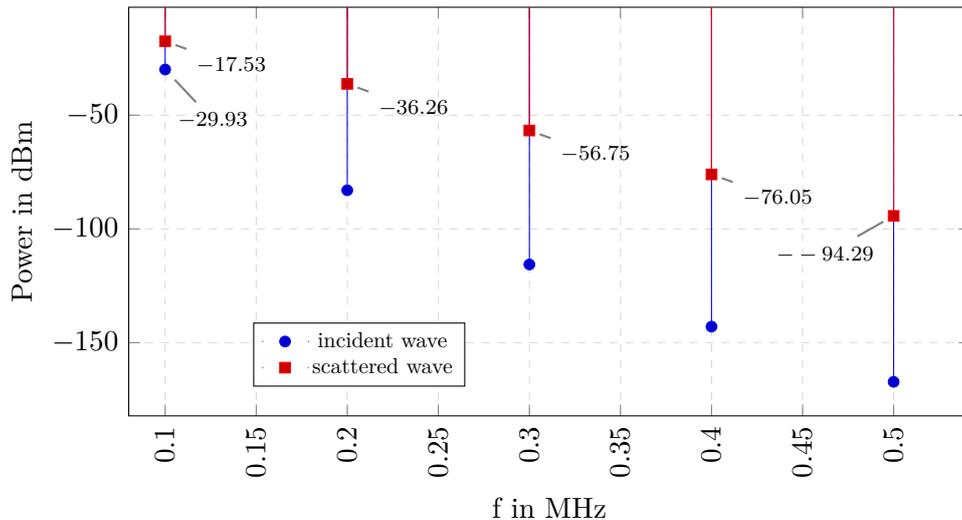


Figure 7: Analysis of the Power of the Scattered and Incident Waves using Harmonic Balance and -30 dBm Power at the Fundamental of 100 kHz

In addition, the powers of the higher harmonics of the scattered waves can be determined from Figure 8. Since we only feed the fundamental tone into the circuit, the power gains over the -30 dBm at the input should coincide with the $X^{(F)}$ parameters of the X-parameter model. These can be found for all relevant harmonics in Figure 8. The full frequency range of

those data is provided in Figure 9.

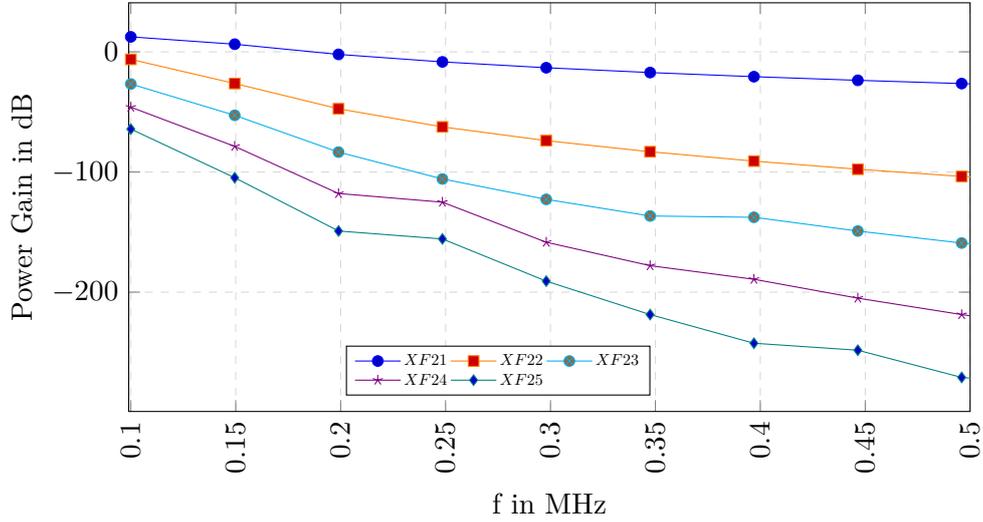


Figure 8: X-Parameter Model, Influence of the Fundamental Harmonic at the Input on the Harmonics at the Output, zoomed at low Frequency Range

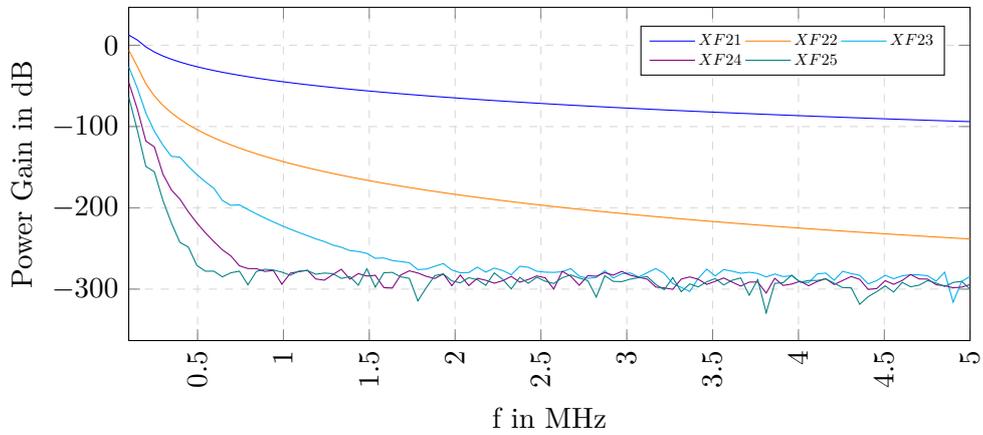


Figure 9: X-Parameter Model, Influence of the Fundamental Harmonic at the Input on the Harmonics at the Output

2.4 Visualizing X-Parameters in AWR DE

Finally, it shall be discussed how the insights gained from the PHD model generator can be utilized, interpreted and displayed in the AWR design environment. For this purpose, it has to be mentioned that AWR provides a

separate component for X-parameter models, the so-called XPARAM block. This can be parameterized using existing XnP files, but is also created automatically after calling the PHD model generator. Since all relevant information is already encoded within the XnP file, only a few parameters can be modified in this block. Namely, only the reference XnP file, the component ID and the number of ports (whereby an error message is generated if this does not match the port definition in the XnP file).

The port assignment of an XPARAM block is in principle not limited, but for a harmonic balance analysis using an APLAC HB simulator (which is required for all non-linear measurements on the component) a harmonic balance port is recommended. This essentially contains a power supply with a source resistor. From here on, one needs to be careful when parametrizing the AWR Design Environment project itself. Namely, the fundamental frequency of the harmonic balance analysis is based on the global frequency settings of the project, but can be overwritten locally. Thereby always the lowest defined frequency is assumed as fundamental frequency. The harmonic balance port should also be parametrized under this aspect. In practice this means that one can not only specify the power of the HB port source at the fundamental frequency, but also add higher harmonics for multi-rate techniques. For simple models, however, this is not necessary for the time being. In addition, the input power of the HB port should also be set to match the input power used to extract the X parameters. This is given in the XnP file and can be extracted easily. If other power parameters are used, deviations from the original model may occur (in the best case only a scaling factor, in the worst case a completely different component behaviour). An example setup with which Figure 3 has been generated is given in Figure 10. Since it has been generated based on an S-parameter file, no matching circuits are required.

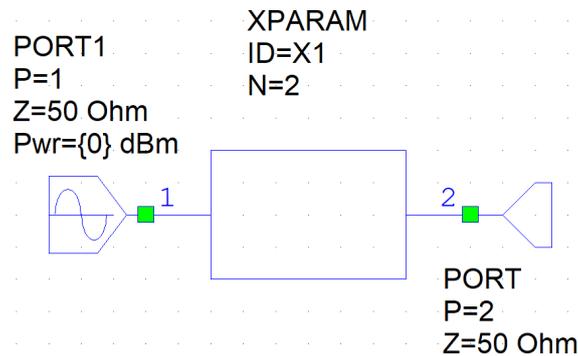


Figure 10: Measurement Schematic for a simple Harmonic Balance Analysis of a X-Paramater Block

It is merely a behavioural model where port 1 is a HB port and port 2 only contains a load resistance matched with the source resistance of port 1 respectively the XnP resistances. Having appropriate schematic provided, the parametrization of the measurement and finally the visualization (both go together in AWR) can now be specified. As should already be familiar, a graph must first be created in the AWR design environment. A simple Cartesian graph (referred to in AWR Design Environment as *Rectangular Graph*) is sufficient for the validation of the PHD model for the time being. Measurements can now be added to this graph. Since X-parameter models are always components or modules with non-linear behaviour, non-linear measurements are required. Here the user has got certain flexibility. It is up to the user whether to measure voltages and currents at the harmonics (which ultimately results in the power quantities) or to determine only the S-parameters under large signal conditions (LSSnm) for the test case of a linear bandpass filter considered in the interim report. An example measurement is given in Figure 11. The result of this measurement can be found in Figure 3 ($X_{11_{20}}$ or $X_{11_{50}}$ depending on the resolution of the X-parameter model used).

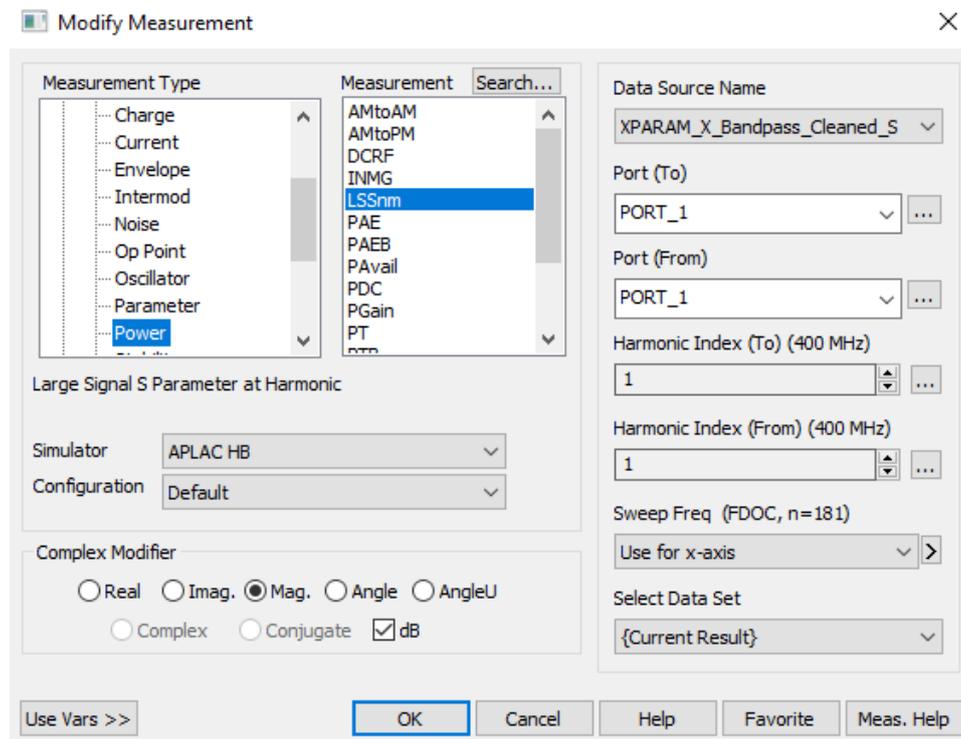


Figure 11: Measurement of the Large-Signal S-Parameters of a X-Parameter Model of a linear Bandpass

3 Conclusion

Based on the knowledge gained during the preparation of this report, it can be confirmed that the PHD model generator can easily convert linear and non-linear models into equivalent X-parameter models. A measurement setup was also identified, which will be extended by multi-rate methods, if required. However, these have yet to be evaluated. In addition, algorithms for nonlinear distortion analysis were developed independently of the AWR Design Environment, which allow the extraction of X-parameter models from the in-house simulator LinzFrame. Results will be reported in another paper.

Whether time-domain methods or the HB are used depends on the problem of investigation. In principle, the time-domain methods are simpler to be implemented. However, distributed elements are naturally modeled in the frequency domain. On the other hand, non-linear time domain devices such as semiconductors are more easily described in time domain.

4 Appendix

4.1 Generalized Impedance Matching

The impedance matching discussed in Section 2.3 is considered here in more detail. A general approach to determining the appropriate passive matching circuits is discussed, based on the well known A-parameters of a linear quadripole. Let

$$\begin{pmatrix} U_1 \\ I_1 \end{pmatrix} = A \begin{pmatrix} U_2 \\ I_2 \end{pmatrix} \quad (5)$$

where A is a 2×2 matrix with coefficients a_{ij} describing the quadripole, U_1, I_1 the input and U_2, I_2 the output voltages and currents. Some examples are provided in Figure 12. It is also worth mentioning that for serially connected quadripole networks, only the A matrices have to be multiplied. Thus, the following conditions are valid for the examples in Figure 12

$$A : A_Y = \begin{pmatrix} 1 & 0 \\ Y & 1 \end{pmatrix} \quad (6)$$

$$B : A_Z = \begin{pmatrix} 1 & Z \\ 0 & 1 \end{pmatrix}$$

$$C : A = A_Y \cdot A_Z = \begin{pmatrix} 1 & Z \\ Y & (1 + YZ) \end{pmatrix}$$

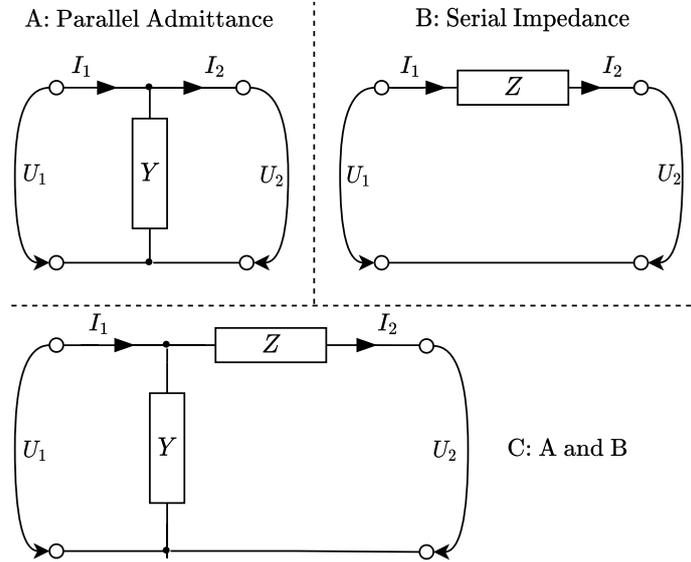


Figure 12: Generalized Formulation of a four-pole Network with parallel Admittances and series Impedances

If an arbitrary load impedance Z_L is connected to port 2 and we want to match it to a given source impedance Z_S , the following equation must be fulfilled

$$\frac{U_1}{I_1} = \frac{U_2 + ZI_2}{YU_2 + (1 + YZ)I_2}, \quad Z_L = \frac{U_2}{I_2}, \quad Z_S = \frac{U_1}{I_1} \quad (7)$$

which may be reformulated as

$$Z_S = \frac{Z_L + Z}{Z_L Y + YZ + 1} \quad (8)$$

However, the problem is under-determined, since Y and Z are the unknowns. There are several options possible. One is to choose the product YZ a-priori. The most practical method is probably to use cheap or available components Z or Y and then to calculate the other component value. It is also possible to use Z to suppress any capacitive or inductive behaviour of the load in advance and to use Y only to adjust a real impedance. An evaluation of the best possibilities is beyond the scope of this appendix. Matlab scripts with suggestions for matching arbitrary load impedances are available upon request.

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